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Dynamical chiral symmetry breaking in strangelets at finite temperature

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1 Introduction

Strangelet[1] has been a hot topic since it was conjectured that strange quark matter might be the absolutely stable state [2, 3]. It is expected that strangelets with low masses might be detected in heavy ion collisions[4], where the ejected particles are formed in hot environment. Searching for strangelets in terrestrial laboratories is important not only for understanding the strong interaction physics, but also because that strangelets are regarded as the unambiguously detected signals for Quark Gluon Plasma(QGP) production[5, 6]. So it is valuable to study the properties of strangelets at finite temperature.

Strange quark matter may not be absolutely stable when realistic current quark masses are introduced which has its origin in the spontaneous breaking of chiral symmetry[7]. Chiral symmetry restoration is enhanced by the finite size effects for droplets with relatively small baryon numbers[8]. Therefore the chiral symmetry breaking is expected to have great influences on the stability and bulk properties of strangelets. In the present paper, the chiral symmetry breaking in strangelets at finite temperature is investigated.

Strangelets at both zero and finite temperature have been frequently studied in the framework of MIT bag model, and many extraordinarily important results were presented[9, 10, 11, 12, 13, 14] in the last decades. However, the essential supposition of the MIT bag model is that quarks are dealt with as free Fermi gas and thereafter their masses take their current masses[15]. Therefore, the dynamical chiral symmetry breaking is absent in the MIT bag model. In Refs.[16, 17, 18], the NJL model is adopted to study the dynamical chiral symmetry breaking in the strangelets at zero temperature, and it is found that the chiral symmetry would break spontaneously at some critical radius. For strangelets, finite size effect should be considered and the multiple reflection expansion (MRE) approximation[19] helps to settle this problem well for strangelets with large baryon numbers. While the shell effect is vitally

important for strangelets with small baryon numbers and the MRE approximation would lead to considerable error[13]. Recently, Yasui brought forward a NJL+MIT bag model[20, 21], in which the NJL model was adopted to describe quark fields inside strangelets and calculations involving mode filling in spherical MIT bag were carried out. The NJL+MIT bag model respects the shell effect and quark masses in the model are dynamically generated. So the model is suitable for studying the dynamical chiral symmetry breaking in strangelets with small baryon numbers at zero temperature.

In the present work, we extend the NJL+MIT bag model in quantum statistical approach to study the strangelets at finite temperature. We construct a grand canonical partition function using the discrete eigenenergies of quarks and the energy contributions from vacuums are taken into account. In addition, the t'Hooft term, which models the $U(1)_A$ symmetry breaking of QCD and has been neglected in Yasui's model, is also considered in the present NJL Lagrangian. In the present model, we focus on the strangelets at β equilibrium only, that means the chemical potentials for quarks with three flavors are equal because the electrons would not exist in strangelets (because the de Broglie wave length of the electron is larger than the size of strangelets[17]). However, it will be straightforward to extend our model to describe strangelets being not in β equilibrium, which are likely to be produced in heavy ion collisions.

2 Model

The strangelet is described by a static spherical MIT bag with radius R , in which valence quarks are confined. The NJL Lagrangian with t'Hooft term[22] is used to describe the quark fields inside the bag. As in the MIT bag model, a surface interacting term \mathcal{L}_δ is introduced to ensure that the vector quark current is continuous on the surface[23]. With these assumptions, the total Lagrangian density is:

$$\mathcal{L}_{QD} = [\bar{q} (i\gamma^\mu \partial_\mu - \hat{m}_0) q + \mathcal{L}_{\bar{q}q}] \theta(R - r) + \mathcal{L}_\delta, \quad (1)$$

where

$$\begin{aligned} \mathcal{L}_{\bar{q}q} = & G \sum_{a=0}^8 [(\bar{q} \tau_a q)^2 + (\bar{q} i \gamma_5 \tau_a q)^2] \\ & - K \{ \det_f [\bar{q}(1 + \gamma_5)q] + \det_f [\bar{q}(1 - \gamma_5)q] \} \end{aligned} \quad (2)$$

and

$$\mathcal{L}_\delta = -\frac{1}{2} \bar{q} q \delta(r - R). \quad (3)$$

And $\hat{m}_0 = \text{diag}(m_{0u}, m_{0d}, m_{0s})$ is the current quark mass matrix in flavor space; the step function θ is to confine the quarks inside the MIT bag. It can be seen that when $R \rightarrow +\infty$, the Lagrangian of (1) reduces to the form that describes quark matter in bulk.

The surface interacting term \mathcal{L}_δ breaks the chiral symmetry explicitly, and in principle a pion cloud coupling with the bag surface could be introduced to recover the broken chiral symmetry. However, it is found that the pion cloud contribution is negligible for strangelets with baryon number $A \gtrsim 5$ [21]. In the following discussions, we will focus on strangelets with baryon numbers of the order of 100, so the present treatment of the surface interacting term is justified.

In the present work, we restrict ourselves to mean-field approximation and focus on the chiral condensates defined as

$$\phi_f = \langle \bar{q}_f q_f \rangle, \quad f = u, d, s. \quad (4)$$

After bosonization, one obtains the linearized version of the model in the mean-field approximation,

$$\begin{aligned} \mathcal{L}_{QD} = & \left[\bar{q} (\text{i}\gamma^\mu \partial_\mu - \hat{m}) q - 2G \sum_f \phi_f^2 + 4K \phi_u \phi_d \phi_s \right] \\ & \times \theta(R - r) - \frac{1}{2} \bar{q} q \delta(r - R), \end{aligned} \quad (5)$$

where the constituent quark mass matrix is

$$\hat{m} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}, \quad (6)$$

in which

$$\begin{aligned} m_i &= m_{0i} - 4G\phi_i + 2K\phi_j\phi_k, \\ (i, j, k) &= \text{any permutation of } (u, d, s). \end{aligned} \quad (7)$$

Then the field equation for $r < R$ and boundary condition at $r = R$ can be derived from (5) by variational principle, they are:

$$\begin{cases} (\text{i}\gamma^\mu \partial_\mu - \hat{m}) q = 0, & r < R, \\ -\vec{n} \cdot \vec{\gamma} q = q, & r = R. \end{cases} \quad (8)$$

Solve the Eq.(8) in spherical coordinates, the eigenstates for quarks with flavor f are obtained

$$q_f(\vec{r}) = \mathcal{N} \left(\epsilon \frac{\text{i}p}{E + m_f} j_{l+\epsilon}(pr) \vec{\sigma} \cdot \vec{n} \right) \mathcal{Y}_{jm}^l(\vec{n}). \quad (9)$$

The meanings of quantum numbers are: the orbital angular momentum l ; the total angular momentum $j = l + \epsilon/2$ ($\epsilon = \pm 1$); the third component projection m of the total angular momentum and the eigenmomentum p . For convenience, we'll use α to represent the quantum number set (l, j, m) in the following. And $\vec{n} \equiv \vec{r}/|\vec{r}|$, \mathcal{N} is the normalization constant, j_l is the spherical Bessel function of rank l , \mathcal{Y}_{jm}^l is the eigenstates of the total angular momentum, $E \equiv \sqrt{p^2 + m_f^2}$.

The eigenstates (9) are for both positive energy states and negative energy states. In positive energy states, eigenmomentum p should satisfy the boundary condition

$$j_l(pR) = \epsilon \frac{p}{E + m_f} j_{l+\epsilon}(pR), \quad (10)$$

while the boundary condition in negative energy states is

$$j_{l+\epsilon}(pR) = -\epsilon \frac{p}{E + m_f} j_l(pR). \quad (11)$$

By introducing the node quantum number n , we can then denote the solutions for (10) and (11) as $p_{f,\alpha}^n$ and $\bar{p}_{f,\alpha}^n$ respectively.

Then, the energies for quarks with flavor f in positive energy states (f, α, n) and negative energy states (f, α, n) are:

$$E_{f,\alpha}(p_{f,\alpha}^n) = \sqrt{p_{f,\alpha}^n{}^2 + m_f^2}, \quad (12)$$

$$\bar{E}_{f,\alpha}(\bar{p}_{f,\alpha}^n) = -\sqrt{\bar{p}_{f,\alpha}^n{}^2 + m_f^2}. \quad (13)$$

In addition, the energy for hole states (antiquarks) $(\bar{f}, \bar{\alpha}, \bar{n})$ which would be excited at finite temperature is:

$$E_{\bar{f},\bar{\alpha}}(p_{\bar{f},\bar{\alpha}}^{\bar{n}}) = -\bar{E}_{f,\alpha}(\bar{p}_{f,\alpha}^n) = \sqrt{\bar{p}_{f,\alpha}^n{}^2 + m_f^2}. \quad (14)$$

Now, we have solved out the single particle (SP) states for quarks inside the bag. In the following, we'll construct the partition function for quarks and antiquarks in terms of SP energies within the particle number representation.

The particle number operators in positive energy states and hole states are denoted as $\hat{N}_{f,\alpha,n}$ and $\hat{N}_{\bar{f},\bar{\alpha},\bar{n}}$, respectively. The Hamiltonian for a strangelet is

$$\begin{aligned} \hat{H} = & \sum_{f,\alpha,n} \nu \hat{N}_{f,\alpha,n} E_{f,\alpha}(p_{f,\alpha}^n) + \sum_{\bar{f},\bar{\alpha},\bar{n}} \nu \hat{N}_{\bar{f},\bar{\alpha},\bar{n}} E_{\bar{f},\bar{\alpha}}(p_{\bar{f},\bar{\alpha}}^{\bar{n}}) \\ & + E_{\text{sea}} + V_{\text{mean}} - E_{\text{vac}}, \end{aligned} \quad (15)$$

where $\nu = 3$ is the degenerate degree in color space; E_{sea} is the energy of the confined vacuum fulfilled by sea quarks in the bag, which is

$$E_{\text{sea}} = \sum_{f,\alpha,n}^{\text{sea}} \nu \bar{E}_{f,\alpha}(\bar{p}_{f,\alpha}^n); \quad (16)$$

V_{mean} is the potential energy brought by the mean fields and its form takes

$$V_{\text{mean}} = \frac{4}{3}\pi R^3 \left(2G \sum_f \phi_f^2 - 4K \phi_u \phi_d \phi_s \right). \quad (17)$$

We have subtracted $E_{\text{vac}} (=4\pi R^3 \varepsilon_{\text{vac}}/3)$ from the Hamiltonian because the measured energy density is the difference between the total energy density of the strangelet and the energy density of the vacuum where the strangelet is absent. E_{vac} is obtained by the NJL model for quark matter in bulk (taking the limit $R \rightarrow +\infty$ in Eq.(1)).

Then the partition function can be constructed as

$$\begin{aligned} \Xi &= \text{Tr} \exp \left[-\beta(\hat{H} - \mu \hat{N}) \right] \\ &= \text{Tr} \exp \left\{ -\beta \left[\hat{H} - \mu \left(\sum_{f,\alpha,n} \nu \hat{N}_{f,\alpha,n} - \sum_{\bar{f},\bar{\alpha},\bar{n}} \nu \hat{N}_{\bar{f},\bar{\alpha},\bar{n}} \right) \right] \right\} \\ &= \exp [-\beta (E_{\text{sea}} + V_{\text{mean}} - E_{\text{vac}})] \\ &\quad \times \prod_{f,\alpha,n} \left\{ 1 + \exp \left[-\beta (E_{f,\alpha}(p_{f,\alpha}^n) - \mu) \right] \right\}^\nu \\ &\quad \times \prod_{\bar{f},\bar{\alpha},\bar{n}} \left\{ 1 + \exp \left[-\beta (E_{\bar{f},\bar{\alpha}}(p_{\bar{f},\bar{\alpha}}^{\bar{n}}) + \mu) \right] \right\}^\nu, \end{aligned} \quad (18)$$

and

$$\begin{aligned} \ln \Xi &= -\beta (E_{\text{sea}} + V_{\text{mean}} - E_{\text{vac}}) \\ &\quad + \sum_{f,\alpha,n} \nu \left\{ 1 + \exp \left[-\beta (E_{f,\alpha}(p_{f,\alpha}^n) - \mu) \right] \right\} \\ &\quad + \sum_{\bar{f},\bar{\alpha},\bar{n}} \nu \left\{ 1 + \exp \left[-\beta (E_{\bar{f},\bar{\alpha}}(p_{\bar{f},\bar{\alpha}}^{\bar{n}}) + \mu) \right] \right\}, \end{aligned} \quad (19)$$

where $\beta = 1/T$ and T is the temperature.

So the free energy of a strangelet is:

$$F = -\frac{1}{\beta} \ln \Xi + \mu N. \quad (20)$$

where N is the total quark number, which satisfies

$$N = \frac{1}{\beta} \frac{\partial}{\partial \mu} \ln \Xi. \quad (21)$$

For thermodynamical stable states, the free energy should take its minimum. So the order parameters ϕ_f of dynamical chiral symmetry breaking should be solved by minimizing the free energy, i.e.

$$\frac{\partial F}{\partial \phi_f} = 0, \quad f = (u, d, s). \quad (22)$$

For stable strangelets in vacuum the pressure should be zero. This condition is used to determine the bag radius and it means

$$\mathcal{P} = - \left[\frac{\partial}{\partial V} F(T, R) \right]_{T, N} = 0, \quad (23)$$

where $V = 4\pi R^3/3$ is the volume of a strangelet.

The NJL model is nonrenormalizable, so a cutoff in momentum space is often used to regularize the infinite summation of SP states. In the present model, we take the form as that in Ref.[20]

$$g(p/\Lambda) = \frac{1}{1 + (p/\Lambda)^\lambda}. \quad (24)$$

Then in numerical calculations, the following substitutions should be made:

$$\begin{aligned} \sum_{f,\alpha,n} &\longrightarrow \sum_{f,\alpha,n} g(p_{f,\alpha}^n/\Lambda), \\ \sum_{f,\alpha,n}^{\text{sea}} &\longrightarrow \sum_{f,\alpha,n}^{\text{sea}} g(\bar{p}_{f,\alpha}^n/\Lambda), \\ \sum_{\bar{f},\bar{\alpha},\bar{n}} &\longrightarrow \sum_{\bar{f},\bar{\alpha},\bar{n}} g(p_{\bar{f},\bar{\alpha}}^{\bar{n}}/\Lambda). \end{aligned}$$

There are 6 parameters in our model totally. Five of them, such as the current quark masses m_{0s} and m_{0q} , couplings G and K and the cutoff Λ , have been determined in Ref.[24] previously. They are: $m_{0q}=0.0055\text{GeV}$, $m_{0s}=0.1409\text{GeV}$, $\Lambda=0.6028\text{GeV}$, $G\Lambda^2=1.803$, $K\Lambda^5=12.93$. This parameter set reproduces the meson masses $m_\pi = 134.98\text{MeV}$, $m_K = 497.65\text{MeV}$ and $m_{\eta'} = 957.78\text{MeV}$ and the π decay constant $f_\pi = 92.2\text{MeV}$. We follow the method in Ref.[20] to adjust the diffuseness parameter λ in Eq.(24) to fit the baryons' mass, $(m_N + m_\Delta)/2 = 1.1\text{GeV}$, in vacuum. We get $\lambda = 27$. Finally, the value of the energy density of the vacuum in bulk ε_{vac} is fixed once the model parameters are determined, and here $\varepsilon_{\text{vac}} = -4.416\text{GeV/fm}^3$.

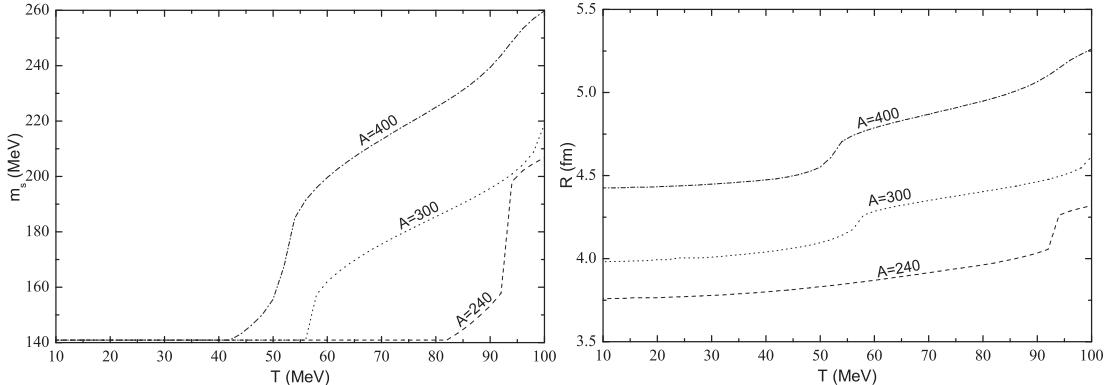


Figure 1: The left panel is the mass of strange quark as a function of temperature; the right panel gives the relation between radii of strangelets with fixed masses and temperature. The three curves in both panels are for strangelets with baryon numbers of 240, 300 and 400, respectively.

3 Results and Discussions

We focus on the dynamical chiral symmetry breaking, which means that the chiral symmetry breaks spontaneously, at finite temperature. We find that the spontaneous chiral symmetry breaking is restored inside the strangelets with baryon number $A \lesssim 150$ when $T \lesssim 100$ MeV, so the quark masses for such strangelets take their current mass values. We take the strangelets with total baryon numbers $A = 240, 300$ and 400 as numerical examples here.

The strange quark masses within strangelets are plotted in the left panel of FIG. 1. Since the condensations ϕ_u and ϕ_d vanish when $T \leq 100$ MeV, the effective masses of nonstrange quarks take their current masses and then we are not going to plot them here. We can see that the chiral symmetries break spontaneously inside the strangelets with $A = 240, 300$ and 400 at critical temperatures $T \simeq 83$ MeV, 57 MeV and 43 MeV respectively, then the strange quarks start to gain their masses by self-energy. And the effective masses of strange quarks begin to increase as the temperature rises, which means that the chiral symmetry will break to a larger extent as the temperature rises. And from the three curves of different baryon numbers, we can see that the chiral symmetries breaks to a larger extent, which is reflected by a larger strange quark mass at a given temperature, inside the strangelets with larger baryon numbers.

Whereas in the standard conjectured QCD phase diagram[25], the spontaneous breaking of chiral symmetry inside the nuclear matter in bulk tends to restore as temperature increases, where there is no external constraints imposed (like the pressure) and the finite size effect is absent. Here we have seen that the strange quark mass relating to the spontaneous breaking of chiral symmetry becomes larger as the temperature increases for the strangelets with zero pressure. It seems abnormal.

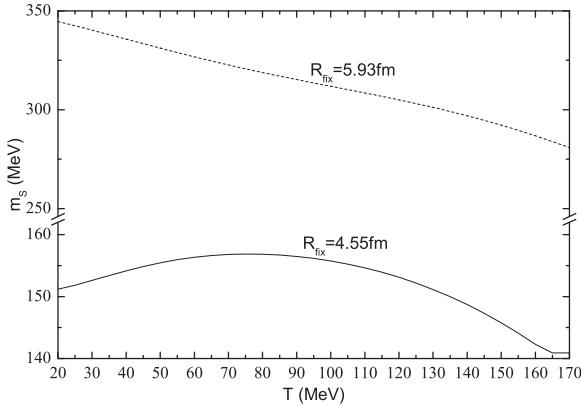


Figure 2: The effective strange quark mass as a function of temperature with the different fixed radius of the bag. The baryon number of the strangelet is taken as 400.

From the radii of strangelets in the right panel of FIG. 1, we can see that the volume of a strangelet is expanding as the temperature increases to keep the pressure at zero. Especially, there is a steep rise of the radius near the critical temperature. Because that the baryon number is conserved, the volume expanding leads to the decreasing of density inside the strangelets. In the standard conjectured QCD phase diagram, the chiral symmetry tends to spontaneously break to a larger extent as the density decrease. Therefore, we conclude that one reason for the abnormal phenomenon is the volume expanding of the strangelets. To corroborate this, we fix the radius of the strangelet artificially at 4.55fm (it is the stable radius at temperature 50MeV) for the strangelet with $A = 400$ and the results are plotted with a solid line in FIG. 2. We can see that the strange quark mass decreases when $T \gtrsim 75\text{MeV}$, and it will eventually decrease to its current mass value at higher temperature. Therefore, our argument is approved.

But we can see that the strange quark mass still increases slowly as the temperature rises for low temperatures (when $T \lesssim 75\text{MeV}$). This reveals that the dynamical chiral symmetry breaking is also affected by the finite size effect of the strangelet, which is usually explained as surface energy and curvature energy in the MRE approximation. In order to illustrate the second reason, we increase the fixed radius of the strangelet. Because the Lagrangian density of strangelet (\mathcal{L}_{QD} in Eq.(1)) will gradually reduce to the form that describes quark matter in bulk as the radius R increases, it is imaginable that the discrete distribution of eigenstates in momentum space will gradually reduce to the continuous distribution as in bulk quark matter when the radius increases, then the finite size effect will become less important correspondingly. The mass of strange quark inside the strangelet with a larger fixed radius 5.93fm (it is 30GeV^{-1} in natural unit) are plotted in FIG. 2 with a dashed line. It can

be seen that the abnormal phenomenon that the strange quark mass increases as the temperature rises vanishes now. And it is also expected that the strange quark mass will eventually decrease to its current mass value at sufficiently high temperature in this case. In conclusion, the strange quark mass inside the strangelets increases as the temperature rises is caused by two reasons: 1) the volume expanding and 2) the finite size effect.

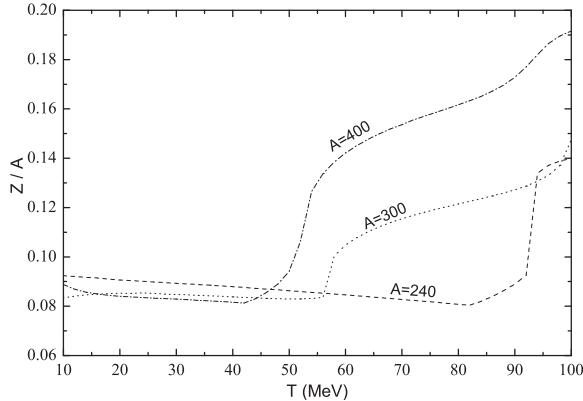


Figure 3: Charge-mass ratios as functions of temperature for strangelets with different masses.

The chiral symmetry breaking would affect the bulk properties of strangelets, one of which is the charge-mass ratio Z/A , i.e. the proportion between the carried charge and the total baryon number of a strangelet. The value of charge-mass ratio is one of the most import physical quantity for distinguishing strangelets in experiments[26]. The charge-mass ratios as functions of temperature are plotted in FIG. 3. We can see that before the spontaneous chiral symmetry breaking happens, the charge-mass ratio is ~ 0.1 , which is the same as in MIT bag model. While after the chiral symmetry breaks spontaneously, the charge-mass ratio begins to increase and it reaches the value of ~ 0.2 at the temperature 100MeV for the strangelet with $A = 400$.

4 Summary and conclusions

Considering a NJL type interaction for quarks confined in a MIT bag, we build a model to describe the strangelets at finite temperature, in which quark masses are dynamically generated. We study the dynamical chiral symmetry breaking which relates to the effective quark masses inside strangelets at finite temperature. It is found that when temperature is under 100MeV, there is no spontaneous breaking of chiral symmetry inside the strangelets with baryon numbers $A < 150$. By taking the strangelets with baryon numbers of 240, 300 and 400 as numerical examples, we find that: 1) the chiral symmetry breaks spontaneously at some finite temperatures

and the strange quark mass will increase as temperature rises; 2) the dynamical chiral symmetry breaking leads to a considerable change of the charge-mass ratio of the strangelets. We illustrate the “abnormal” phenomenon that the strange quark mass inside the strangelets increases as temperature rises by two reasons: 1) the volume expanding and 2) the finite size effect. Financial support by the National

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